4.2. Confirmation of the map-improvement principle

After the described formalization, we can experimentally verify the map-improvement principle. We use concrete medieval and modern maps whose ordering is free of doubt. Here, we indicate the most interesting of them: (1) the map of the world by Cosmas Indicopleustes, 6th c. A.D. (see [13], V. 1, p. 20, Fig. 11); (2) plane map by Cosmas Indicopleustes, 6th c. A.D. (*ibid.*); (3) arabic map by al-Istakhrī 950 A.D. (*ibid.* V. 3, p. 221, Fig. 45); (4) map by Macrobius of the 10–15th cc. A.D. ([249], p. 85, Fig. 9); (5) map of the 11th c. in the Cottonian collection from the British Museum (*ibid.* V. 3, p. 223, Fig. 47); (6) map of the 12th c. A.D. from the Turin Library ([13] V. 2, p. 300, Fig. 111); (7) several European maps of the 14th c. in the History Museum, Moscow; (8) map from the 15th-c. book *Opus sphericum* by Sacro Bosco; (9) map of the world of 1470, the so-called *Rad Karte* ([273], p. 13); (10) a map of the world by Stefano Borgia of the 15th c. ([13], V. 2, p. 633, Fig. 162); (11) 6th-c. plane map of the world, representing the terrestrial globe, by Johannes Stabius (Stabius–Dürer–Karte, 1515) ([273], p. 15); (12) map of the 16th-c. book *Mũndialis Sphere Opusculũm* by Sacro Bosco of 1519; (13) map by T. Occupario of 1522 (from the History Museum, Moscow); (14) map of the world by Diego Ribeiro of 1527, ([273], p. 14); (15) map of Cornelius Niccolai of 1598 (from the History Museum, Moscow); (16) terrestrial globe of the 17th c. (from the History Museum, Moscow); and (17) several modern maps.

The map-improvement principle was absolutely verified against this, not very considerable, data, and the averaged graph of $L_{\text{aver}}(T)$ practically coincided with the theoretical one in Figs. 39 and 42. In particular, it means that the above relative order of medieval maps was generally chronologically correct.

Hence, a method follows for finding a chronologically correct order of a collection of maps whose datings are unknown or doubtful, for which we first enumerate the maps under investigation in an arbitrary order, and construct the associated matrix $L\{T\}$, i.e., all the graphs of $L(T_0, T)$. We then start mixing up the maps, i.e., change their relative order by means of all possible permutations σ, each time computing the matrix $L\{\sigma T\}$ associated with the permutations, and strive for reducing the matrix to the ideal and theoretical form (see Figs. 39 and 42). This ordering of the maps for which the matrix is closest to the theoretical, and the graph of $L_{\text{aver}}(T)$ is monotonically decreasing, should be taken as chronologically correct and required. The fact that the map-improvement principle was confirmed permits us also to offer a method for dating old maps. Let A be a certain map whose dating is unknown. Construct its map-code, and subjoin it to the map-code collection of the maps already dated. Construct the graphs of $L(T_0, T)$ and the matrix $L\{T\}$ for all the maps of the collection, and assign A its number T_0. In accordance with the above procedure, we find the chronologically correct order for the whole collection. In particular, we find a place for A, which permits us to date the map with respect to the other dated maps. The method was applied to the following series of old maps: (1) the well-known map from the *Geography* of Ptolemy (edition of 1545; see [249], p. 97, Fig. 11), traditionally related to the 1st–2nd cc. A.D. fell into the 15th–16th cc. A.D., near maps 8–15 from the above list; (2) the famous *Tabula pentingeriana* ([13], V. 3, pp. 232–233, Fig. 48), traditionally related to the time of Augustus Octavian fell