We can determine the histogram $f_2(j) = P_A(\xi_2 = j)$ by means of the square name matrix $K_{n\times n}$ associated with the given list [24]. Therefore, the formula is valid:

$$f_2(j) = \begin{cases} \dfrac{2}{N^2}\displaystyle\sum_{i=1}^{n}\sum_{s=i}^{n-j} K(i, s+j)K(i,s), & 0 \leqslant j \leqslant n-1, \\ \dfrac{1}{N^2}\displaystyle\sum_{i=1}^{n}\sum_{s=1}^{n} K(i,s)^2, & j = 0, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $K(\cdot,\cdot)$ are the elements of $K_{n\times n}$.

Formula (2) follows directly from the definition of the random variable ξ_2 and $K(i,s)$ being the total of the multiple names from the set of those "born" in X_i, which get into the chapter X_s.

The square matrix is insufficient for the construction of f_3. Therefore, we have to resort to a rectangular name matrix supplying complete information regarding chapters of the list (see [21]).

5. Histograms related to the name and nationality lists of Roman popes. We now discuss the construction of the histograms f_1 and f_2, related to the lists Π and H of well-known popes and their nationalities from A.D. 50 (Peter) until the present day (see, e.g., [119]). Characteristically, the names or nationalities have no explicit succession in these lists. Accordingly, there are good grounds to believe that Statement (B) should be fulfilled if the above lists are chronologically correct. Note that if we do assume the existence of a succession, then a hypothetically correct chronology can only explain the splash near the origin on the histograms f_2 and f_3 (see Item 6).

We divided Π and H into 10-year long chapters, the lists' length being $N = 293$, the number of chapters $n = 190$, and that of the different names $k = 87$. We made use of the rectangular and square matrices constructed from Π and H by A. A. Makarov.

We found by direct computation that the histogram $f_1(j)$ for Π and H is, to a very high accuracy, a linear decreasing function for $j = 1, \ldots, n-1$. See the form of f_2 in Figs. 75, 76. On the abscissa, the values of the scatterings were recalculated into years.

It can be seen that f_2 for Π possesses a series of sharp splashes. According to the above argument, we can single out the following groups of shifts for f_2 and Π, viz.,

(i) by 40–50 and (doubling it) 80–100 years,

(ii) by 300 and 330–350 years,

(iii) the group of 11 consecutive shifts separated by *c.* 100 years by: 400, 480, 580, 670, 760, 850, 940, 1,050, 1,140, 1,230 years,

and

(iv) by 1,400 years.