

are not sufficient to determine reliable group errors for neighbourhoods of only two stars: Aquila and Sirius. This was the reason why we excluded them from further consideration.

## 8. The Dating of the Almagest Star Catalogue

8.1. *Statistical dating procedure.* Let  $I$  be the set of eight named stars (see above) and

$$\Delta(t, \gamma, \varphi) = \max_{i \in I} |\Delta b_i(t, \gamma, \varphi)|.$$

We base our dating procedure on the hypothesis that the latitudes of *all* named stars from  $I$  must have *individual* errors of not more than  $10'$  in the year  $t^*$  of the observations. In other words,

$$\Delta(t^*, \gamma, \varphi) \leq 10',$$

and the value  $\gamma$  belongs to the statistical tolerance interval (see Fig. 116). Corresponding adoptable values of  $\gamma$  are "marked" by points in Fig. 116. Consequently, we claim that the time interval  $600 \text{ A.D.} \leq t^* \leq 1300 \text{ A.D.}$  can be considered as a dating interval. Of course, this interval depends on different parameters in general: claiming accuracy ( $10'$ ), confidence probability  $\epsilon$ , and some others. The stability of the method will be analysed in Section 9 below.

8.2. *Geometrical dating procedure.* Though we have determined a dating interval, some doubts about it can appear due to the statistical nature of some of the assertions. In reality, we based our assertions on the fact that group errors for neighbourhoods of the eight named stars are the same. This fact was proved with the help of statistics. Hence, there is some positive probability (though it is very small) that this fact is wrong.

Let us again consider the value  $\Delta(t, \gamma, \varphi)$  and find for every  $t$  the quantities

$$(\gamma_{\text{geom}}(t), \varphi_{\text{geom}}(t)) = \arg \min_{\gamma, \varphi} \Delta(t, \gamma, \varphi)$$

and

$$\Delta_{\text{min}}(t) = \Delta(t, \gamma_{\text{geom}}(t), \varphi_{\text{geom}}(t)).$$

These quantities depend only on the position of the eight named stars, whereas  $\hat{\gamma}_{\text{Zod } A}(t)$  and  $\hat{\varphi}_{\text{Zod } A}(t)$  do not depend on them (they depend on the position of all stars in  $\text{Zod } A$ ). It is clear that  $\Delta_{\text{min}}(t) \leq 10'$  if  $600 \text{ A.D.} \leq t \leq 1300 \text{ A.D.}$  But it turned out that  $\Delta_{\text{min}}(t) \leq 10'$  if and only if  $600 \text{ A.D.} \leq t \leq 1300 \text{ A.D.}$  (see Fig. 119). Besides,  $\gamma_{\text{geom}}(t) \approx \hat{\gamma}_{\text{Zod } A}(t)$  for these  $t$  (see Fig. 116). Hence, this confirms without any statistical arguments that the above interval is a dating interval. There do not exist a  $\gamma$  and  $\varphi$  such that the inequality

$$\Delta(t, \gamma, \varphi) \leq 10'$$

holds when  $t < 600 \text{ A.D.}$  or  $t > 1300 \text{ A.D.}$  We confirmed also that the systematic error calculated with the help of statistics (using the coordinates of all stars in  $\text{Zod } A$ )