histogram $f_1(j) = P(\xi_1 = j)$ *linearly decreases* on the set $j \in \{0, 1, 2, \ldots, n-1\}$, and $f_1(j) = 0$ for $j < 0$ and $j \geqslant n$. In fact, ξ_1 takes the value j in $(n-j)p^2$ cases out of N^2 possible ($|\Omega| = N^2$), since there exist $n - j$ ways of fixing the chapter with the first name; the second chapter is fixed uniquely in accordance with the first one and the number j, whereas the set of names with scattering j is of power p^2. Thus,

$$f_1(j) = P(\xi_1 = j) = (n-j)p^2/N^2 \quad (0 \leqslant j \leqslant n-1).$$

In the sequel, we will always make the list under consideration dense sufficiently uniformly, i.e. the histogram $f_1(j)$ linear with respect to j on the set $\{0, 1, \ldots, n-1\}$. For example, computations show that the condition is immediately fulfilled to a very high accuracy for the list of popes' names.

We can determine the histogram $f_2(j) = P_A(\xi_2 = j)$ by means of the square name matrix $K_{n \times n}$ associated with the given list. Namely, the formula,

$$f_2(j) = \begin{cases} N^{-2} \sum_{i=1}^{n} \sum_{s=i}^{n-j} K(i, s+j) K(i, s) & (0 \leqslant j \leqslant n-1), \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

is valid where $K(.,.)$ are the elements of $K_{n \times n}$. Formula (2) directly follows from the definition of the random variable ξ_2 and $K(i, s)$ being the total of the multiple names from the set of those 'born' in X_i, which get into the chapter X_s.

The square matrix is insufficient for the construction of f_3. Therefore we have to resort to a rectangular name matrix supplying complete information regarding chapters of the list.

10 Numerical experiments with real lists

We now discuss the construction of the histograms f_1 and f_2, related to the well-known popes' name and nationality lists P and H from A.D. 50 (Peter) until the present day (Bickerman, 1968; Blair, 1882; Gregorovius, 1900–1909; Lozinsky, 1934; Morozov, 1926–1932; Rumyantsev, 1936). Characteristically, the names or nationalities have no explicit succession. Accordingly, there are good grounds to believe that Statement 2 should be fulfilled if the above lists are chronologically correct. Note that if we do assume the existence of a succession, then a hypothetically correct chronology can only explain the splash peak near to the origin on the histograms f_2 and f_3 (see below).

We divided P and H into 10-year long chapters, the length of the list being $N = 293$, the number of chapters $n = 190$, and the number of different names $k = 87$. We made use of rectangular and square matrices constructed from P and H by Dr. A.A. Makarov. We found by direct computation that the histogram $f_1(j)$ for P and H is, to a very high accuracy, a linearly decreasing function for $j = 0, 1, \ldots, n-1$, and zero for other j. See the form of f_2 in Fig. 7. On the axis of abscissas, the values of scatterings were recalculated into years. It can be seen that f_2 for P, in Fig. 7(a), possesses a series of sharp splashes. According to the above argument, we can single out the following shift groups for f_2 and P, namely:

(i) through 40–50 and (doubling it) 80–100 years;
(ii) through 300 and 330–350 years;
(iii) the group of 11 consecutive shifts separated by approximately 100 years through: 400, 480, 580, 670, 760, 850, 940, 1050, 1140, 1230 years;
(iv) through 1400 years.